Calculus as ganita

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Abstract

The course aims to teach calculus as it developed in India as ganita, since that makes calculus very easy, hence enables students to solve harder real-life problems not covered in usual calculus courses, as has been demonstrated in 5 universities in 3 countries.[1] It will cover all three aspects of history, philosophy, and techniques related to calculus.

Calculus originated in India[2] with the 5th c. Aryabhata, who used finite differences and a recursive technique to numerically solve differential equations, to derive precise trigonometric values.[3] A key difference from existing calculus[4] or mathematical analysis[5] or numerical analysis courses is that the current course reverts to the original: instead of formal real numbers and limits, this new course uses the *avyakta ganita* of the 7th c. Brahmagupta's polynomials.[6] Brahmagupta's polynomial arithmetic is better-known today as arithmetic in the non-Archimedean field of rational functions.[7]

Non-Archimedean arithmetic, as in Robinson's non-standard analysis, admits infinitesimals, which Newton tried but failed to understand through his doctrine of fluxions. Just because Newton's doctrine of fluxions was widely regarded as incomprehensible[8] and non-rigorous by Europeans, formal real numbers, R, were invented by Dedekind[9] in 1870. Axiomatically speaking, R is algebraically characterized as the largest ordered field with the so-called Archimedean property which makes infinitesimals impossible in R. But since *R is an ordered field but a proper extension of R, hence, the Archimedean property fails in the non-standard reals *R; the consequent existence of infinitesimals in *R makes (unique) limits impossible. While the use of non-standard analysis and its infinitesimals was long ago proposed as a way to make calculus easy to teach,[10] this failed because the starting point, viz. non-standard analysis, is impossibly hard[11] and beyond even most professional mathematicians.

The present course overcomes this difficulty by reverting to Brahmagupta's original *avyakt ganita* (polynomial arithmetic is taught in schools) as the reason for the failure of the so-called Archimedean property. The Western failure to understand the calculus imported from India is very similar to the Western failure for 9 centuries to understand elementary arithmetic ("Arabic numerals") also imported from India starting with the 10th c. Gerbert[12] (pope Sylvester II) and the 13th c. Fibonacci.[13] This failure persisted till the 19th c, with Euler,[14] Pascal[15] and the 19th c. de Morgan[16] making foolish statements about integers, e.g. that $-9 < 0$ is impossible. How could those who failed to understand negative numbers (i.e., ordering in integers) hope to understand ordering in polynomial arithmetic? And why should one admit the Western unsubstantiated claim to have evolved a "superior" understanding of calculus? Many public challenges to public debate that claim have been persistently dodged, and it is just authoritatively enforced through censorship[17] etc.

In calculus as ganita, the use of non-Archimedean arithmetic is combined with the philosophy of zeroism,[18] or inexactitude, which involves discarding or zeroing of infinitesimals and small quantities, for example to sum infinite series, as in the sum of an infinite geometric series first given by Nilakantha.[19] The course will also explain that the European belief in the exactitude of mathematics, a belief absent in ganita, arose from Plato's religious belief in mathematics as mathesis.[20] This involved the use of mathematics, regarded as "eternal truth", as a superstitious technique of

sympathetic magic to arouse the eternal soul. However, all practical applications of mathematics, such as sending a rocket to the moon, anyway involve approximate calculations. Furthermore, these applications are today done on high-speed computers which use floating point-numbers, NOT real numbers. And floats do not even obey the associative law for addition.[21] However, instead of accepting this as proving the failure of exactitude, it is amusing how people either conflate the two, reals and floats, or assert that floats which deliver practical value are erroneous while axiomatic real numbers which have no practical value are "correct".

The third aspect which the course will emphasize is the philosophy of ganita which accepts both the empirical (प्रत्यक्ष प्रमाण) and deductive inference as a means of proof, exactly as does science. This differs from the present-day philosophy of axiomatic math which prohibits[22] the use of the empirical in proof. It is false, and a mere superstition,[23] that axiomatic proof is hence "superior" since either infallible or less fallible than proofs which involve the empirical. In fact, axiomatic proofs are MORE fallible. Further, the use of axiomatically-proved mathematical theorems may lead science astray, for there is no empirical means to verify the metaphysical axioms (of, say, axiomatic set theory), hence no way to be sure that axiomatically proved theorems are valid knowledge in reality. It is also false[24] that there are any actual axiomatic proofs in the Euclid book, but such a myth was created when the book first arrived in Europe from Arabs ca. 1125 during the Crusades. Axiomatic proofs were falsely read into the book, but first used in the Crusading church theology of Aquinas[25] as a political trick to enable padres to arrive at any convenient conclusions in theology. In mathematics, they were used for the first time by Hilbert in 1899,[26] and I have issued various public challenge-prizes for any evidence to the contrary.[27] It is astonishing that the current philosophy of mathematics has to be founded on such brazen lies about axiomatic proof in "Euclid" taught to children.

The Indian way to do calculus is also ideally suited for implementation on present-day high-speed computers, and my free software CALCODE would be used for this course.

Compared to usual university calculus courses, this new course on calculus as ganita offers several advantages. (1) It leads to greater conceptual clarity (compared to formal real numbers and limits which students and even teachers rarely understand correctly). The reason for lack of understanding is that the starting point of axiomatic real numbers involves axiomatic set theory, which few understand (as distinct from naïve set theory). In acknowledgment of this widespread difficulty in teaching calculus, California recently cancelled the teaching of calculus in schools.[28] (2) The new course is far easier than the usual dreaded calculus courses,[29] since it simplifies calculus by using infinitesimals, as in nonstandard analysis, but uses only the essential principle of non-Archimedean arithmetic from Brahmagupta's *avyakta ganita*, thus avoids the complexities of non-standard analysis, since what is important is practical value of calculus, not adherence to standard analysis. (3) It hence enables students to solve much harder real-life problems not covered in usual calculus courses, such as non-elementary elliptic integrals. (4) Unlike usual calculus courses, real practical applications to physics and engineering are solved as actually done today. (5) The usual calculus course emphasizes the "skill" of symbolic manipulation (tricks to calculate derivatives and integrals ONLY of elementary functions) but those tricks are inconsequential today, given existing open-sources software for symbolic manipulation such as MAXIMA which would be taught as part of the course, just to emphasize the uselessness of learning those tricks to integrate/differentiate elementary functions, a major part of calculus exams today.

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[1] For a quick summary see the recent presentations for (1) an invited talk at the 9th Pacific Rim Conference in Mathematics, Darwin, Australia, https://ckraju.net/papers/presentations/points-ckr-australia-calculus-talk.html, and (2) a colloquium at IIT Mandi, https://ckraju.net/papers/presentations/iit-mandi-plain.html. See, also, this reading list on decolonisation: https://tinyurl.com/decol-list-new.

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